The formulæ (1) and (2) have been obtained without making any assumption as to the configuration of the comparison stars. Moreover, by basing the calculation directly on the errors of the coordinates of the comparison stars, instead of using the probable errors of the plate constants, the necessity of assuming an absence of correlation between these constants has been avoided.

The most important inference is that the object required should be at, or very near, the origin—*i.e.* should coincide as closely as possible with the centroid of the comparison stars. If this condition be satisfied, the configuration of the stars and the method of reduction (by four or six constants) are practically immaterial. The probable errors, so far as they are due to erroneous constants, are given by

$$p_x = p_y = p / \sqrt{n}$$

If the object does not coincide with the centroid of the comparison stars it is desirable to make the moments of inertia about both axes in the plate large—that is, to have the stars widely scattered in both directions—if the six-constant solution is used; but if the four-constant solution is used it is only necessary that one of the principal moments should be large. The four-constant solution is more valuable when the solution is to apply to the whole extent of the plate, particularly when the distribution of the comparison stars is bad.*

University Observatory, Oxford:
1904 *May* 12.

Note on the formulæ connecting "Standard Coordinates" with Right Ascension and Declination. By F. W. Dyson, M.A., F.R.S.

The simplest and most interesting formulæ from a mathematical point of view for determining Standard Coordinates are those given by M. Trépied in the Introduction to the Algiers Section of the *Astrographic Catalogue* (p. v), and all others are readily deduced from them.

* Mr. Filon, to whom I have had the advantage of showing the above note in MS., points out the desirability of stating definitely the assumptions which underlie the argument—namely, that "there is no correlation between the errors in the residuals of two given stars," and also that "there is no correlation between errors of measurement in the two coordinates" for each star. He adds that "distortion or ellipticity of the images may correlate the errors in x, y; and, further, that if this distortion affect similarly all the star discs, it may introduce correlation into the x-measures of different stars." But in general, in work of the kind to which this note is intended to apply, the distortion of the images can be considered so slight that no serious effect of this nature need be apprehended.

Let x and y be the Standard Coordinates α, δ; the Right Ascension and Declination of a star; and A, D the Right Ascension and Declination of the centre of the photograph :

$$\left.\begin{aligned} \text{Then}\quad \sqrt{1+x^2+y^2}\sin\delta &= \sin D + y\cos D \\ \sqrt{1+x^2+y^2}\cos\delta\sin(\alpha-A) &= x \\ \sqrt{1+x^2+y^2}\cos\delta\cos(\alpha-A) &= \cos D - y\sin D \end{aligned}\right\} \quad \begin{matrix}\text{i}\\ \text{ii}\\ \text{iii}\end{matrix}$$

The second of these formulæ gives

$$x = \sin(\alpha-A)\cos\delta + \tfrac{1}{2}x(x^2+y^2) + \&c.$$

If $x = y = 1°$

$$\tfrac{1}{2}x(x^2+y^2) = 1''\cdot 09.$$

Thus the term $\frac{1}{2}x(x^2+y^2)$ may be readily tabulated as a small correction, or a diagram may be formed to give it. It is to be noted that $\frac{1}{2}x(x^2+y^2)$ is independent of the position of the plate's centre and very rough values of x and y are sufficient to give the correction accurately.

The first equation gives y in the terms of the declination. When $x = 0$ the equation becomes

$$\sqrt{1+y^2}\sin\delta = \sin D + y\cos D$$

and as $y = \tan(\delta - D)$ is known to be a solution of this equation, the following transformation of equation (i) is suggested.

$$\{y - \tan(\delta-D)\}\{y+\tan(\delta+D)\} = \frac{x^2\sin^2\delta}{\cos(\delta-D)\cos(\delta-D)},$$

$$\text{or}\quad y - \tan(\delta-D) = \frac{\tfrac{1}{2}x^2\tan\delta}{1+\tfrac{1}{2}\,\dfrac{y-\tan(\delta-D)}{\tan\delta}\cdot\dfrac{\cos(\delta-D)\cos(\delta+D)}{\cos^2\delta}}$$

$$\left.\begin{aligned} &\text{For brevity put } y - \tan(\delta-D) = \tfrac{1}{2}x^2\tan\delta\,.\,z \\ &\text{and}\quad \frac{\cos(\delta-D)\cos(\delta+D)}{\cos^2\delta} = \lambda^2 \end{aligned}\right\}$$

Then $z\,(1+\frac{1}{4}\lambda^2x^2\,.\,z) = 1$

$$\text{or}\quad z = 2\,\frac{\sqrt{1+\lambda^2x^2}-1}{\lambda^2x^2}$$

$$= 1 - \frac{\lambda^2x^2}{4} + \frac{1\cdot 3}{4\cdot 6}\lambda^4x^4\,\frac{1\cdot 3\cdot 5}{4\cdot 6\cdot 8}\lambda^6x^6 + \&c.$$

Therefore

$$y = \tan(\delta - D) + \tfrac{1}{2}x^2 \tan \delta - \tfrac{1}{8}x^4 \tan \delta \frac{\cos(\delta - D)\cos(\delta + D)}{\cos^2\delta}$$
$$+ \tfrac{1}{16}x^6 \tan \delta \frac{\cos^2(\delta - D)\cos^2(\delta + D)}{\cos^2\delta} - \&c.$$

This formula shows explicitly the magnitude of the correction at different declinations to the approximate formula

$$y = \tan(\delta - D) + \tfrac{1}{2}x^2 \tan \delta.$$

For example, take the case where $\delta = 85°$, $D = 84°$, and $x = 1°$.

$$-\tfrac{1}{8}x^4 \tan \delta \frac{\cos(\delta - D)\cos(\delta + D)}{\cos^2\delta}$$
$$= +\tfrac{1}{8}\frac{\sin^3 1° \,.\, \cos 1° \,.\, \cos 11°}{\sin^3 5°} \times 3600'' = 3''{\cdot}55.$$

For $\delta = 70°$ the correction can only amount to $0''{\cdot}06$.
This correction can be readily tabulated as far as $\delta = 85°$.

An Analysis of the Distribution of Stars on the 1,180 Plates in Zones +25° to +31° allotted to the University Observatory, Oxford, in connection with the International Astrographic Survey. By F. A. Bellamy.

1. The completion, on 1904 February 17, of the measurement in two positions of the 1,180 plates required for the zones +25° to +31°, which were allotted to the University Observatory, Oxford, in connection with the International Astrographic Catalogue of Stars to the eleventh magnitude, has afforded the means of revising the figures given by me in a paper (*Monthly Notices*, vol. lx. p. 12) concerning the distribution of stars. It is my intention now only to refer to that portion of the sky to which the resources of the observatory have been mainly devoted during the past twelve years, reserving consideration of the distribution of stars outside +25° to +31° to a future communication.

The plates discussed in the paper referred to were 513 in number, each with exposures of 6^m, 3^m, 20^s, and were exposed between 1892 January and 1899 June; other plates with exposures not precisely of these durations were omitted. In the present paper all plates are included, but only one of the three exposures has been dealt with throughout.

2. In selecting plates considered good, and rejecting others which did not show enough stars, constant use has been made of